Optimum Design Of Thin-Walled Box Beams with Coupled Bending and Torsion Using Frequency Constraints

Ramana V. Grandhi* and Jamshid K. Moradmand†
Wright State University, Dayton, Ohio

Introduction

A LARGE percentage of optimization papers dealing with the beam finite elements consider cross sections where the centroid and the shear center coincide. This is true for some structural applications, but a number of structures such as turbine, helicopter and propeller blades, and automotive and aerospace structures can have cross sections where the inertial axes do not coincide with the elastic axes. This results in coupling between some of the bending and torsional modes. In the present work, optimum structural design of vibrating beams with noncollinear inertial and elastic axes is considered. The condition of noncollinear axes exists in structures having unsymmetric cross sections. The beam behaves as a coupled elastic system where the vibration modes involve simultaneous bending and torsional displacements. The natural frequencies of the coupled vibrations are different from the natural frequencies computed either for pure bending or pure torsion. In this paper, optimum design of vibrating beams with thin-walled rectangular box cross sections having one axis of symmetry is considered. This results in coupling of one of the bending vibrations with the torisonal displacements, whereas the other bending vibrations occur independently. In the following, some of the previous design studies on coupled bending and torsion vibrations are reviewed.

Peters et al.[1] considered the helicopter blade optimization for desired placement of natural frequencies. The authors considered symmetric cross sections in obtaining the optimum stiffness and mass distributions. Recent work by Chattopadhyay et al.[2] and Hanagud et al.[3] developed an optimality criterion method for the design of thin-walled channel beams with one axis of symmetry. The minimization of the structural volume with a single frequency constraint and its dual problem of maximization of the fundamental frequency with a volume constraint were considered for the simply supported and cantilever boundary conditions. In Ref. 4, a mathematical programming technique based on an extended quadratic interior penalty function method was employed for the design of vibrating beams with a channel section. In this paper, a rectangular box cross section applicable to a typical blade design (see Fig. 1) is studied. Optimum structural design was done using up to three natural frequency constraints. In addition to the structural mass, nonstructural masses were added on the structure. Optimum flange thickness distribution, natural frequencies, and design iteration histories are presented for the cantilever and simply supported end conditions.

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*Associate Professor, Department of Mechanical and Materials Engineering.

†Graduate Student, Department of Mechanical and Materials Engineering.

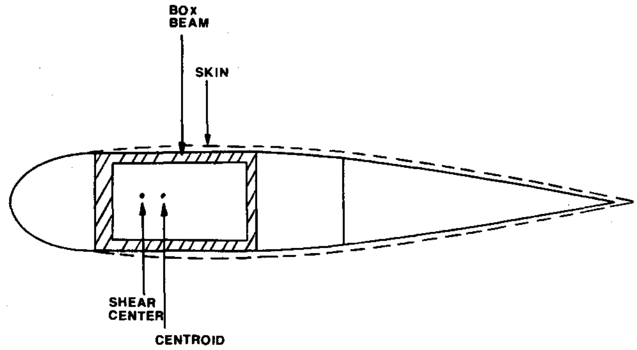

Fig. 1 Blade cross section.

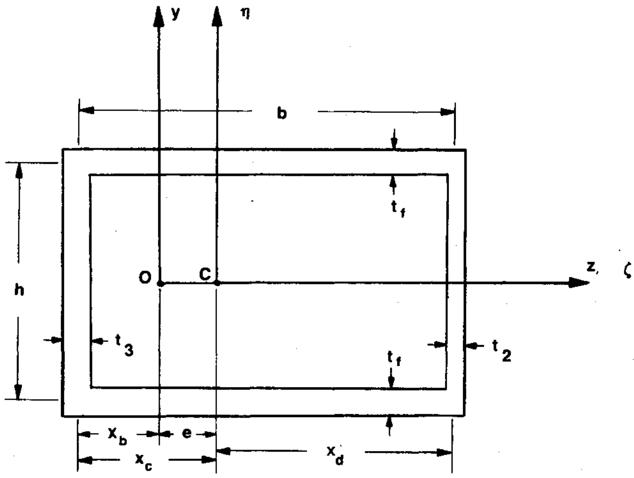

Fig. 2 Box cross section.

Finite Element Formulation

Beams with rectangular box cross sections with one axis of symmetry shown in Fig. 2 are studied in this paper. In Fig. 2, b is the width of the cross section, h is the height of the cross section, t_2,t_3 are web thicknesses, t_f is flange thickness, X_b is the distance between the shear center and the center of web, and X_c is the distance between the centroid and the center of web. Bending vibrations in the y direction are coupled with the torsional vibrations; whereas vibrations in the z direction occur independently. The beam structure is divided into a number of finite elements. Each node possesses six degrees of freedom (θ, v, w, $\mathrm{d}\theta/\mathrm{d}x$, $\mathrm{d}w/\mathrm{d}x$, $\mathrm{d}v/\mathrm{d}x$): the angle of twist, the displacement in the y direction, the displacement in the z direction, the rate of change of the angle of twist, the rate of change of z-direction displacement, and the rate of change of y-direction displacement, respectively.

The element stiffness and mass matrices of the beam undergoing coupled vibrations are derived from the strain energy U and the kinetic energy T of the beam element, respectively.

$$U=\frac{EI_\zeta}{2}\int_0^l\left(\frac{\partial^2 v}{\partial x^2}\right)^2\mathrm{d}x+\frac{EI_\eta}{2}\int_0^l\left(\frac{\partial^2 w}{\partial x^2}\right)^2\mathrm{d}x+\frac{GJ}{2}\int_0^l\left(\frac{\partial\theta}{\partial x}\right)^2\mathrm{d}x \tag{1}$$

$$T=\frac{\rho A}{2}\int_0^l\dot{v}^2\mathrm{d}x+\frac{\rho A}{2}\int_0^l\dot{w}^2\mathrm{d}x+\rho Ae\int_0^l\dot{v}\dot{\theta}\mathrm{d}x+\frac{\rho I_0}{2}\int_0^l\dot{\theta}^2\mathrm{d}x \tag{2}$$

where I_ζ,I_η are the centroidal moments of inertia, E is the modulus of elasticity, G is the modulus of rigidity, I_0 is the polar moment of inertia about the shear center, J is the torsional constant, A is the cross-sectional area, and ρ is the mass density.

In Eq. (1) the first term is due to bending in the y direction, the second term is due to bending in the z direction, and the third term is due to torsion. For a rectangular box cross section, the cross section will twist and warp under the action of the shear loads. Unlike the case of the beams with an open cross section,[5] the torque resulting from nonuniformity of warping is small compared to the St. Venant's torque. Hence, the strain energy associated with warping of the cross section is neglected. In Eq. (2) the first term is due to bending in the y direction, the second term is due to bending in the z direction, the third term is due to the coupling of y-direction bending and torsion, and the fourth term is due to torsion.

Problem Statement

In this paper, the following two optimization problems were considered:

1) Minimization of structural volume subject to multiple frequency constraints

$$g_j(\mathbf{x}) = \omega_j - \tilde{\omega}_j \geq 0 \qquad j = 1,2,\ldots,m \tag{3}$$

2) Maximization of the fundamental frequency with a specified volume constraint

$$g(\mathbf{x}) = V(\mathbf{x}) - \tilde{V} = 0 \tag{4}$$

and also side constraints

$$x_i \geq x_i^l \qquad i = 1,2,\ldots,n \tag{5}$$

where $\tilde{\omega}_j$ and $\tilde{V}$ represent frequency and volume limits, respectively, and superscript l represents the lower limit on the design variable. The above stated optimization problems were solved using a quadratic extended interior penalty function formulation and modified Newton's method of unconstrained minimization. The general purpose optimization algorithm NEWSUMT-A[6] was used for this purpose. This algorithm employs approximate Hessian matrix using the first-order derivatives of the constraint functions. The one-dimensional search is carried using a Golden Section method. NEWSUMT-A handles equality and inequality types of constraints. In this work, the objective function and constraint gradients with respect to the design variables were calculated using a finite difference scheme.

Optimization Results

A thin-walled beam of rectangular box cross section with one axis of symmetry (see Fig. 2) was considered for optimization studies. The 40-in.-length beam was modeled with 10 equal-length finite elements with possibly differing values of flange thicknesses. Flange thicknesses were considered as design variables. All other cross-sectional dimensions were preassigned. The structural material was aluminum with Young's modulus of 10^7 psi, rigidity modulus of 3.8×10^6 psi, and density of 0.243×10^{-3} lb $-$ s^2/in.^{4}. A 0.02 = lb nonstructural mass was added at each free node. The initial design variables were 0.025 in. and the lower bound on the design variables was 0.007 in. The natural frequencies and mode shapes were computed using the subspace iteration and Jacobi methods. In this work, optimization results are presented for the simply supported and cantilever end conditions. Both end condition results are discussed separately because of different thickness distributions realized at the optimum.

Simply Supported Beam

First, the volume of the simply supported beam was minimized with single and multiple frequency constraints. The structural volume was minimized subject to a constraint of $\omega_1 \geq 200.0$ rad/s. The total volume of the beam was reduced from 2.7 in.^{3} to 2.18 in.^{3}. The optimal flange-thickness distribution followed the pattern of the bending-moment distribution corresponding to the fundamental mode of vibration. The bending-moment distribution corresponding the fundamental mode of vibration is maximum at the center and is minimum at the two ends of the beam. The design iteration history is shown in Fig. 3.

Next, the beam was designed with multiple frequency constraints. The volume minimization problem was posed with three frequency constraints of $\omega_1 \geq 200.0$ rad/s, $\omega_2 \geq 300.0$ rad/s and $\omega_3 \geq 350.0$ rad/s. The total volume of the beam was reduced from 2.7 in.^{3} to 2.34 in.^{3}. The natural frequencies at the optimum design were $\omega_1 = 200$ rad/s, $\omega_2 = 324.664$ rad/s and $\omega_3 = 350.0$ rad/s. The first and third constraints were satisfied as equality constraints even though they were posed as inequality constraints. The optimal flange thickness distribution has maximum at the support and minimum at the center of the beam. This distribution is different from the one observed with the single frequency constraint. The optimal flange thickness distribution in this case is effected by the third mode of vibration. The third mode of vibration is a coupled bending and torsional mode in which the torsion is dominant. Hence, the optimal flange thickness distribution followed the pattern of the twisting moment distribution corresponding to the third mode of vibration.

Next, the simply supported beam was designed for the fundamental frequency maximization with an equality constraint on the total volume of the beam. Thus, the objective was to maximize the fundamental frequency of vibration and to keep the volume unchanged from the initial design. In this case, the

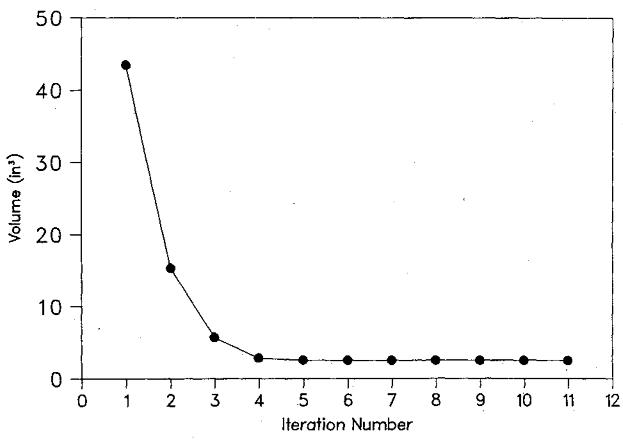

Fig. 3 Simply supported beam volume minimization-iteration history.

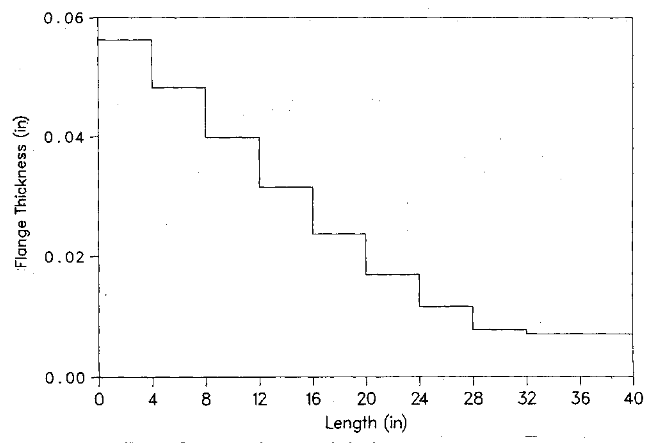

Fig. 4 Cantilever beam volume minimization-flange thickness distribution.

objective function is nonlinear, and the constraint is linear. The fundamental frequency was increased from 212.554 rad/s to 218.225 rad/s. The optimal flange-thickness distribution followed the pattern of the bending-moment distribution corresponding to the fundamental mode of vibration.

Cantilever Beam

Next, cantilever-end conditions were considered. First, the volume of the beam was minimized with single and multiple frequency constraints. The fundamental frequency of the beam at the initial design was 72.465 rad/s. The volume was minimized subject to a fundamental frequency constraint of $\omega_1 \geq 75.0$ rad/s. The total volume of the beam was reduced 33% – from 2.7 in.^{3} at the initial design to 1.80 in.^{3} at the optimum design. The optimal flange-thickness distribution is shown in Fig. 4. The optimal flange-thickness distribution has a maximum at the support with a minimum at the free end of the beam. The optimal flange-thickness distribution followed the pattern of the bending moment distribution corresponding to the fundamental mode of vibration.

Next, the volume of the cantilever beam was minimized subject to multiple frequency constraints. The volume minimization problem was posed with three frequency constraints of $\omega_1 \geq 75.0$ rad/s, $\omega_2 \geq 120.0$ rad/s, and $\omega_3 \geq 140.0$ rad/s. The total volume was reduced to 1.91 in.^{3}. The first three natural frequencies at the optimum design were $\omega_1 = 75.0$ rad/s, $\omega_2 = 122.039$ rad/s, and $\omega_3 = 140.0$ rad/s. The first and third constraints were satisfied as equality constraints even though they were posed as inequality constraints. The optimal flange thickness distribution was similar to the one obtained for the single constraint case.

Finally, the cantilever beam was designed for the fundamental frequency maximization with a constraint on the total volume of the beam. The fundamental frequency was increased from 72.465 rad/s to 99.862 rad/s. The optimal flange thickness distribution assumes a maximum at the support and a minimum at the free end of the beam.

Conclusions

In this paper, optimum design of beams vibrating in coupled bending and torsion was considered. The selection of the lower limit on design variables is important to prevent the switching of the shear center location with respect to the centroid during the progress of optimization. Optimum-thickness distributions and design-iteration histories were presented for the simply supported and cantilever beams with rectangular box cross section. The optimal flange-thickness distribution followed the pattern of the dominant mode of vibration.

Acknowledgment

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References

[1]Peters, D. A., Ko, T., Korn, A., and Rossow, M. P., "Design of Helicopter Rotor Blades for Desired Placement of Natural Frequencies," *Proceedings of the 39th Annual Forum of the American Helicopter Society*, American Helicopter Society, May 1983.

[2]Chattopadhyay, A., Hanagud, S. V., and Smith, C. V., Jr., "Minimum Weight Design of a Structure with Dynamic Constraints and A Coupling of Bending and Torsion," *Proceedings of the AIAA/ASME/ASCE/AHS 27th Structures, Structural Dynamics, and Materials Conference*, AIAA-CP-86-0928, Part 2, AIAA, New York, 1986, pp. 386-394.

[3]Hanagud, S., Smith, C. V., Jr., and Chattopadhyay, A., "Optimal Design of a Vibrating Beam with Coupled Bending and Torsion," *AIAA Journal*, Vol. 25, No. 9, 1987, pp. 1231-1240.

[4]Grandhi, R. V., and Moradmand, J. K., "Optimum Synthesis of Thin-Walled Vibrating Beams with Coupled Bending and Torsion," *Journal of Mechanisms, Transmissions, and Automation in Design*, Vol. 3, No. 4, 1989, pp. 555-560.

[5]Timoshenko, S. P., and Goodier, N. J., *Theory of Elasticity*, 3rd ed., McGraw-Hill, New York, 1970.

[6]Grandhi, R. V., Thareja, R. T., and Haftka, R. T., "NEWSUMT–A: A General Purpose Program for Constrained Optimization Using Constraint Approximations," *ASME Journal of Mechanisms, Transmissions, and Automation Design*, Vol. 107, 1985, pp. 94-99.

Low-Speed Pressure Distribution on Semi-Infinite Two-Dimensional Bodies with Elliptical Noses

Erik S. Larson*
FFA, The Aeronautical Research Institute of Sweden, Bromma, Sweden

Introduction

IN a recent Note,[1] a semiempirical representation of the pressure distribution on ellipsoids of revolution and elliptic-nosed, semi-infinite bodies of revolution in axial, incompressible flow was constructed by taking the lead from early theoretical results.[2,3] A similar approach is applied to a family of semi-infinite, two-dimensional bodies with elliptical noses in symmetric, incompressible flow, for which Hess and Smith[3] have presented numerical solutions of the potential flow equations. As could be expected, the same type of expressions that appeared as in the axial flow case, after modification of some constants and exponents, gave a very good representation of the theoretical results for nose slenderness ratios up to unity. The pressure drag of the noses was to be determined by numerical integration.

Analytic Expressions

Elliptic Profiles

The analytic expression for the pressure distribution on ellipsoids of revolution in axial, incompressible flow, obtained by Maruhn,[2] from a solution of the potential flow equations, is the same for elliptic profiles. The characteristic number A, specifying the peak velocity ratio, is changed, however. Thus

$$C_p = 1 - A^2 \sin^2\theta = 1 - (V/U_\infty)^2_{\max} \sin^2\theta \tag{1}$$

with

$$A = 1 + \delta \tag{2}$$

where $\delta = b/a$ is the profile slenderness ratio, b and a being, respectively, the minor and the major axes of the elliptic section. The angle θ is

$$\theta = \sin^{-1}\left(\frac{2\xi - \xi^2}{\delta^2 + (1-\delta^2)(2\xi - \xi^2)}\right)^{1/2} \tag{3}$$

being, as shown in the insert sketch in Fig. 1, the angle between the symmetry axis and the normal-to-the-profile contour. In

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*Aerodynamics Department (retired). Member AIAA.